Closing Tues. (Nov. 28th): HW 4.3
Exam 2 is **Tuesday**!!!
covers 3.1-3.6, 3.9-3.10, 10.2, 4.1
Expect: 6 pages
Page 1: Find Deriv./Slope/Tangent
Page 2-4: Implicit, Parametric, Linear Approx., Abs. Max/Min
Page 5-6: Related Rates (Expect to see at least one picture/question directly HW).

Office Hours today: 1:30-3:00 COM B-006

Entry Task:
Find the abs. max and min of
$f(x) = x^3 e^{-x}$ on $[-1,5]$.

4.3 Classifying Critical Points (Local Max/Min)

Recall:

$y = f(x)$	$y' = f'(x)$
horiz. tangent	zero
increasing	positive
decreasing	negative
vertical tangent, sharp corner, or not continuous	does not exist

<u>Key, big, essential observation</u>

(*First derivative test*)

If $x = a$ is a critical number for $f(x)$

AND

if $f'(x)$ changes from…

1. …positive to negative, then a **local maximum** occurs at $x = a$.

2. …negative to positive, then a **local minimum** occurs at $x = a$.

Example: Find and classify the critical numbers for

$$y = x^3 + 3x^2 - 72x$$

Example: Find and classify the critical numbers for

$$y = x^4 - 2x^3$$

Example: Find and classify the critical numbers for

$$y = x^{2/3}$$

Example: Find and classify the critical numbers for

$$y = \frac{x^3}{x^2 - 1}$$

The 2nd Derivative

$$y'' = f''(x) = \frac{d}{dx}(f'(x))$$

= "rate of change of first derivative"

Terminology

If **f''(x) is positive**,
then the **slope of f(x) is increasing**
and we say f(x) is **concave up**.

If **f''(x) is negative**,
then the **slope of f(x) is decreasing**
and we say f(x) is **concave down**.

A point in the domain of the function at which the concavity changes is called an **inflection point**.

Summary:

$\boldsymbol{y = f(x)}$	$\boldsymbol{y'' = f''(x)}$
possible inflection	zero
concave up	Positive
concave down	Negative
possible inflection	does not exist

Example: Find all inflection points and indicate where function is concave up and concave down for

$$y = x^4 - 2x^3$$

Clever Observation

(*Second Derivative Test*)

If $x = a$ is a critical number for $f(x)$

AND

1. if $f''(a)$ is positive (CCU),
   then a local min occurs at $x = a$.

2. if $f''(a)$ is negative (CCU),
   then a local max occurs at $x = a$.

3. if $f''(a) = 0$,
   then we say the 2nd deriv. test is
   inconclusive (need other method)

Example: Find and classify the critical numbers for

$$y = 2 + 2x^2 - x^4$$

(use the 2^{nd} deriv. test)